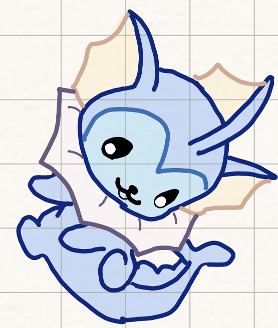
10.25.22

LECTURE 26

Definition

For each t in the domain of r, a vector valued function, the unit tangent vector to C @ t is $T(t)$:

$$T(t) = \frac{r'(t)}{\|r'(t)\|}$$



we also know that $\|T(t)\| = 1$.

Definition

if r is a smooth parameterization of C, T is the unit tangent vector, and $T'(t) \neq 0$, the unit normal vector to C @ t noted $N(t)$ is

$$N(t) = \frac{T'(t)}{\|T'(t)\|}$$

this always points in the inwards direction of concavity

Example

Find unit tangent and unit normal vectors to the curve $r(t) = \langle 1+t, t^2 \rangle$ @ $-1, 0, 1$

tangent: $r'(t) = \langle 1, 2t \rangle$

$\|r'(t)\| = \sqrt{1^2 + 4t^2} = \sqrt{4t^2+1}$ $\quad T(t) = \left\langle \frac{1}{\sqrt{4t^2+1}}, \frac{2t}{\sqrt{4t^2+1}} \right\rangle$

normal: $r''(t) = \langle 0, 2 \rangle$

$\|r'(t)\| = 2$ $\quad N(t) = \langle 0, 1 \rangle$

Observation if $r(s)$ is an arc-length param. of C, then

$$T(s) = r'(s)$$
$$N(s) = \frac{r''(s)}{\|r''(s)\|}$$

Definition the binormal vector to C @ t is

$$B(t) = T(t) \times N(t)$$

$B(t)$, $N(t)$, and $T(t)$ make a mini moving coord system which is called the Fernet Frame

Theorem
$$B(t) = \frac{r'(t) \times r''(t)}{\|r'(t) \times r''(t)\|}$$

If $r(s)$ is the arc length parameterization

$$B(s) = \frac{r'(s) \times r''(s)}{\|r''(s)\|}$$

14.4 Problems + Lecture 26 Probs

14.4 Problems

3) $T(t) = \dfrac{r'(t)}{\|r'(t)\|} = \dfrac{\langle 2t, 1\rangle}{\sqrt{4t^2+1}}$ @ $t=1$

$r'(t) = \langle 2t, 1\rangle$

$$\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$$

$$N(t) = \frac{r''(t)}{\|r''(t)\|} = \frac{\langle 2, 0\rangle}{2} = \langle 1, 0\rangle$$

11) $r = r_0 + tv$ prove that $r = r(t_0) + sT(t_0)$

$r = r_0 + uv$

$$\frac{dr}{du} = v \qquad s = \int_0^t \left\| \frac{dr}{du} \right\| du = t\|v\|$$

$$t = \frac{s}{\|v\|}$$

$r = r_0 + s\left(\frac{v}{\|v\|}\right)$ because $v = \frac{dr}{du}$,

we know that $T(t) = \frac{v}{\|v\|}$

then, substituting $t = t_0$ gives

$$r = r(t_0) + sT(t_0) \ \blacksquare.$$

Lecture 26 Problems

1) $r(t) = \langle t^2 + 2t, 3t\rangle$ $\quad T(t) = \dfrac{\langle 2t+2, 3\rangle}{\sqrt{4t^2+8t+13}}$ @ $t=0$,

$T(0) = \dfrac{\langle 2, 3\rangle}{\sqrt{13}}$ $\quad \boxed{B}$

2) $r(t) = \langle t\sqrt{3}, \sin t + 2, \cos t + 3\rangle$ $\quad B(t) = \dfrac{r'(t) \times r''(t)}{\|r'(t) \times r''(t)\|}$

$r'(t) = \langle \sqrt{3}, \cos t, -\sin t\rangle$

$r''(t) = \langle 1, -\sin t, -\cos t\rangle$

$$\begin{vmatrix} i & j & K \\ \sqrt{3} & \cos t & -\sin t \\ 1 & -\sin t & -\cos t \end{vmatrix} = \hat{i}(-\cos^2 t - \sin^2 t) - \hat{j}(-\sqrt{3}\cos t + \sin t) + \hat{k}(-\sqrt{3}\sin t - \cos t)$$

$$\sqrt{\cos^4 t - 2(1) + \sin^4 t - 3\cos t^4}$$

$\boxed{C}$

3) $r(t) = \langle 2t, e^t, 3t^2 \rangle$ @ $t=0$

$r(0) = \langle 0, 1, 0 \rangle$

$\boxed{D}$

$r'(t) = \langle 2, e^t, 6t \rangle = \langle 2, 1, 0 \rangle$

4) $\boxed{A}$